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Conventional Proportional-Integral Control of a Dividing-Wall Distillation Column with Discrete Measurements

The problem of controlling an energy-efficient dividing-wall distillation column (DWDC) with a discrete key variable measurement is addressed. Usually, controllers are tuned in a systematic and simultaneous way, based on a certain control system configuration and conventional discrete proportional-integral controllers. The same stable pole-assignment approach to linear dynamics is followed here that approximately describes the convergence of each control loop. Tuning relationships in terms of sampling time and predecessor parameters of physical insight are obtained. The control system performance is simulated for disturbance rejection in an energy-efficient DWDC for separating a ternary mixture of benzene, toluene, and xylene. This tuning technique can be effective with a sampling time as long as that corresponding to current measurement instruments.

Keywords: Discrete measurements, Dividing-wall distillation column, Pole-assignment tuning, Proportional integral control, Systematic tuning

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1 Introduction

Distillation is the most used separation process in the chemical industry and one of the steps in any corresponding plant with the highest energy consumption. The effective usage of energy drives research in this field to optimize existing systems or to create novel ones, whose configurations and process conditions depend on the type of mixture to be separated. Often it is assumed that the more complex the distillation system, the more complex the control, or the more complex the dynamic behavior [1, 2]. In any case, distillation systems are sensitive to changes in process conditions [3, 4]. Therefore, once a distillation system has been configured, the subsequent issue is the design of an appropriate control system, mainly in order to evaluate the feasibility of operation.

Following the line of control system design for distillation columns, the majority of works use conventional proportional (P), proportional-integral (PI), and proportional-integral-derivative (PID) controllers, depending on the kind of variable to be controlled, and the challenge focuses on establishing the better control configuration. Aiming on the composition regulation of output streams, in most studies it is assumed that these variables are measured in-line and continuously. Recognizing that in practice these variables are not measured in-line, some works propose control systems in which the temperatures of certain stages are regulated, which in turn regulates the composition of output streams around their nominal values [5, 6]. Furthermore, to improve closeness inference schemes, relationships of composition with respect to tray temperatures have been proposed as well [7].

Although inferential schemes seem to be effective, in a current practical scenario of distillation column operation the composition measurements are typically carried out via sampling and laboratory analysis, where the resulting measurements are discrete. Therefore, controllers are required that are appropriately driven by the actual measurements. Once a control configuration has been established with continuous controllers, the first simple solution lies in the substitution of continuous controllers with corresponding discrete ones. However, the remaining matter is the tuning of controllers in distillation columns that has generally become an experience-based activity [8], in which certain procedures have been proposed [9]. In the first instance, these procedures can be followed for discrete controllers. However, a new degree of freedom must be taken into account: the sampling time.

Recently, the systematic tuning of conventional PI controllers in a dividing-wall distillation column (DWDC) has been addressed [8]. In the above-mentioned study, the researchers followed a stable pole-assignment approach to obtain tuning relationships that even allowed for the simultaneous tuning of all controllers. In this work, the counterpart of tuning discrete

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PI controllers for DWDCs is addressed within the same pole-assignment approach. The aim is to obtain tuning relationships that take into account the effect of sampling time, in addition to well-known (and identifiable) parameters for each control loop (input-output pair), such as static gains and time constants. Also, as a representative case, it is referred to the DWDC control system of Ling and Luyben [10] that characterizes and illustrates the functioning and performance provided by the resulting tuning procedure.

This control study is important since new designs [11] and applications of DWDCs have been reported. For instance, the production and purification of biodiesel in a single DWDC [12–14], and the purification of bioethanol [15–18]. Delgado-Delgado et al. [17] studied the dehydration of bioethanol using a DWDC, and that the energy required in the reboiler depends strongly on the interconnecting flows and the increasing carbon dioxide emissions as the energy needed in the reboiler increases. Also, the cooling water required in the condenser changes in the same way as the energy supplied to the reboiler. Other applications of thermally coupled distillation columns include the purification of biobutanol, detecting the optimal operations in terms of energy consumption and controllability analysis [19].

2 DWDC Control System and Tuning Problem

Here, it is referred to the DWDC control system used to separate a continuous ternary mixture in Zavala-Guzmán et al. [8]. Indeed, the general characteristics are given in accordance with a system originally described by Lin and Luyben [10].

2.1 Dividing-Wall Distillation Column

The process displayed in Fig. 1 consists of a DWDC of N[1]; stages, with a wall that divides stages N_{DS} up to N_{DI}; the first stage is the condenser, and the final stage is the reboiler. At stage N_F, the mixture of composition $z_1/z_2/z_3$ (mol %, in order of volatility) is fed at a rate F_F, temperature T_F, and pressure P_F. The vapor split ratio is S_V, and the liquid split ratio is S_L, according to the fraction of the fluid that goes by the feeding side. The column is operated at a reflux ratio R, a reboiler heat input Q, and a side stream flow F_S, at the intermediate stage N_M on the opposite side of the feeding. The DWDC is equipped with devices that regulate liquid levels in the reboiler, the condenser, and that guarantee a base pressure P_B and temperature T_B. A distillate stream rich in the more volatile component, a side stream rich in the intermediate volatile component, and from the bottom the heaviest component are obtained.

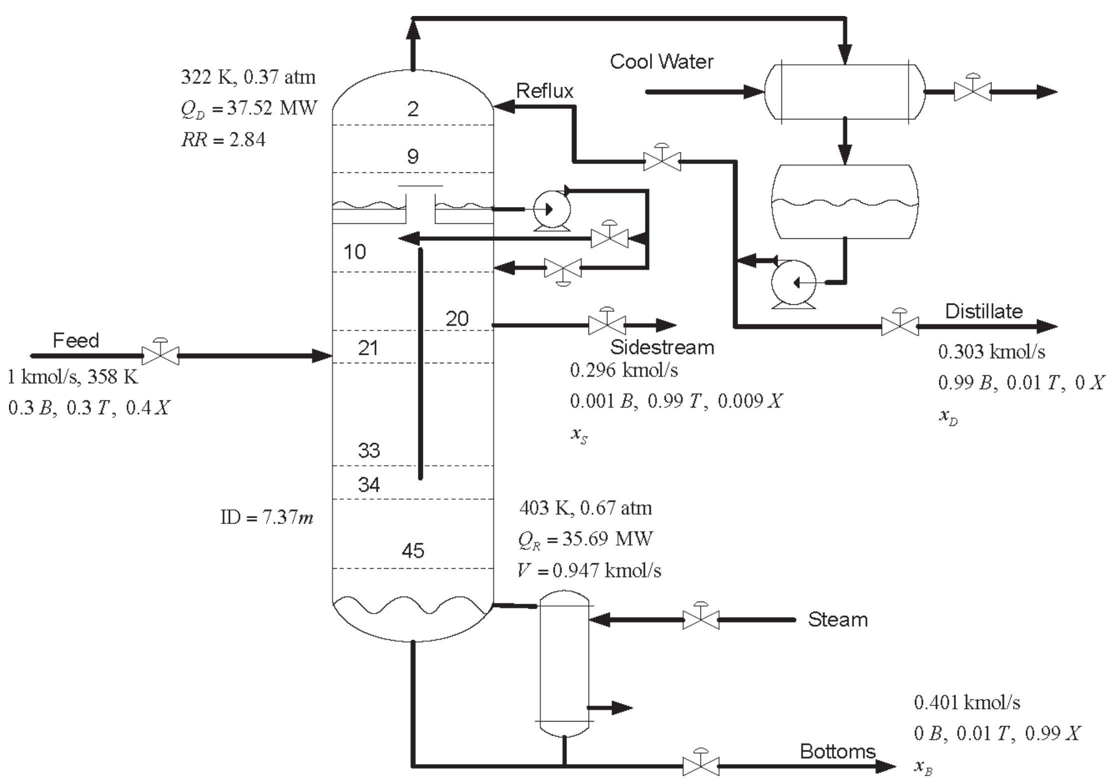

Figure 1. DWDC flowsheet [10].

1) List of symbols at the end of the paper.

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2.2 Control System

Process control is based on impurities such as control variables as follows: y_D is the measurement of the intermediate volatile component composition in the distillate stream (x_D), y_S is one of the least volatile component compositions in the sidestream (x_S), y_B is the intermediate volatile component composition in the bottom stream (x_B), and y_{NS-1} is the composition of the less volatile component at stage N_{S-1} (x_{NS-1}). The control inputs are R, Q, F_S, and S_L. To deal with potential changes in non-manipulated and/or unknown disturbances, such as $z_1/z_2/z_3$ and F_F, the configured control loops are illustrated in Fig. 2 and described in Eq. (1):

$$L_i: (u_j, y_j),\quad L_1: (R, y_D),\quad L_2: (S_L, y_{NS-1}),\quad L_3: (F_S, y_S),\quad L_4: (Q, y_B) \tag{1}$$

The parings in the control loops indicated in Eq. (1) are those reported by Ling and Luyben [10]. Also, it is important to mention that other pairings in the control loops can be considered; e.g., in the work of Mathew et al. [20], regarding the control of an ideal endothermic reactive distillation column with and without heat integration, the composition of the distillate was controlled by manipulating the reflux rate and the bottom's composition was controlled by adjusting a tray temperature in the stripping section. The selection of the pairings in the control loops can be done by using the relative gain array, or this selection can be carried out in terms of exergy analysis [21]. The last option obtains an energy-efficient control structure by incorporating second law efficiency.

2.3 Discrete PI Controllers

According to the discrete nature of the measurements, obtained at certain periodic instants t_k,

$$y_i(t_k) = x_i(t_k),\quad t_{k+1} = t_k + \delta t, \tag{2}$$

$$y = [y_D, y_{NS-1}, y_S, y_B],\ x = [x_D, x_{NS-1}, x_S, x_B],\ u = [R, Q, F_S, S_L], \tag{3}$$

the discrete counterpart of the conventional PI controller is given by:

$$u_i(t) = \bar{u}_i + K_C^i \tilde{y}_i(t_k) + \frac{K_C^i}{\tau_I^i} \sum_{j=0}^{k} \tilde{y}_i(t_j)\delta t, \quad t \in [t_k, t_{k+1}),\quad \tilde{y} = \bar{y} - y \tag{4}$$

where $\bar{y}$ and $\tilde{y}$ are the nominal and deviation values of the variable y, respectively. K_C^i and τ_I^i are the proportional gain and the integral time, respectively, of the controller for the control loop L_i (Eq. (1)).

2.4 Tuning Problem

Having designed the control system, the matter that remains is the tuning of the controllers: there are two gains to determine for each control loop. In the above case, a total of eight gains

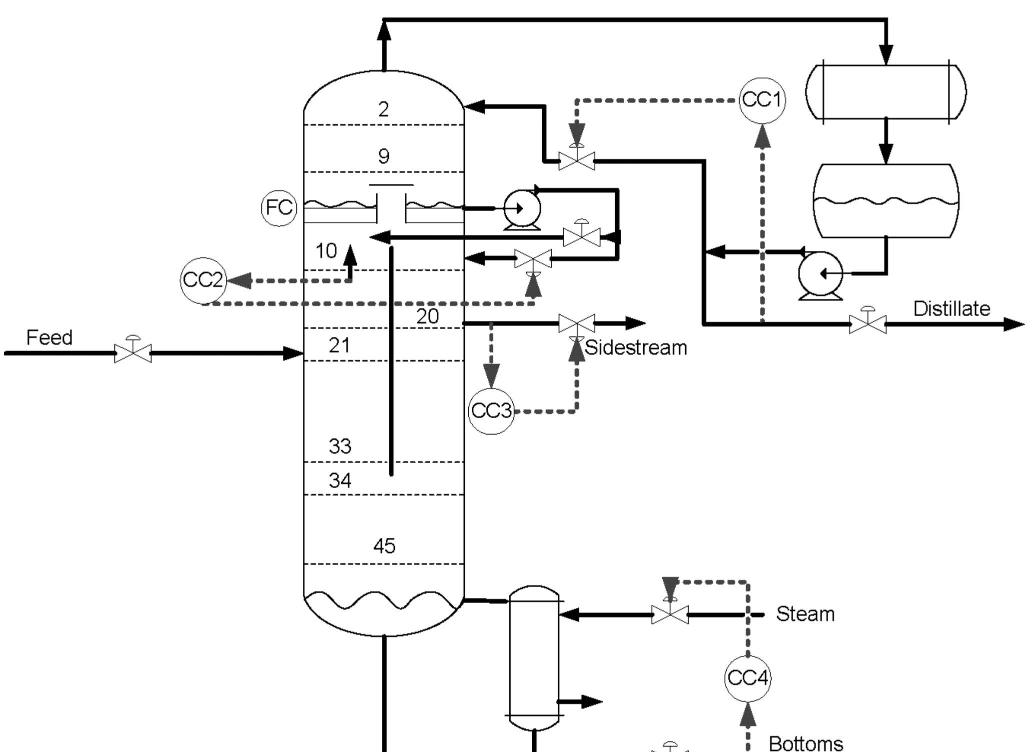

Figure 2. DWDC control system.

result. This generates the problem of developing a systematic tuning technique: in the same way like the continuous counterpart, with a reduced number of tuning buttons driving all the controllers, and that use the simplest information which can be obtained from a process: response time and sensitivity of the process with respect to certain inputs. In this case, according to Eq. (2), there is one additional parameter to take into account: δt.

Thus, the tuning problem consists of obtaining tuning relationships in terms of time constants and static gains, and tuning parameters that drive the convergence behavior of all controllers. It is worth noting that the first step in the method applied here involves a pole-placement approach based on first-order-type dynamics, identified for each control loop that allow taking into account the effect of δt.

3 Development of Tuning Technique

A framework to obtain effective values for controller gains is constructed, which is the discrete counterpart of an early work of Zavala-Guzman et al. [8]. It follows that an assignment of stable poles to the dynamics approximately describes the convergence of certain outputs when a corresponding PI controller is implemented.

The following ordinary differential equation (ODE) approximately describes the response of a certain output with respect to its coupled control input (Eq. (1)):

$$\begin{aligned}&\dot{\tilde{y}}_j(t) = -\frac{1}{\tau_{\mathrm{P}}^j}\tilde{y}_j(t) + \frac{K_{\mathrm{P}}^j}{\tau_{\mathrm{P}}^j}\tilde{u}_j(t),\\ &\tilde{y} = \bar{y} - y, \quad \tilde{u} = \bar{u} - u, \quad j = 1,\ 2,\ 3,\ 4\end{aligned} \tag{5}$$

As a first step facing the discrete nature of controllers, the ODE is the time discretized considering control input as a step-wise function:

$$\tilde{y}_j(t_{k+1}) = \alpha_j\tilde{y}_j(t_k) + \beta_j\tilde{u}_j(t_k), \quad t_{k+1} = t_k + \delta t \tag{6}$$

$$\alpha_j(\delta t, \tau_{\mathrm{P}}^j) = e^{-\frac{1}{\tau_{\mathrm{P}}^j}\delta t}, \quad \beta_j(\delta t, K_{\mathrm{P}}^j, \tau_{\mathrm{P}}^j) = K_{\mathrm{P}}^j\left(1 - e^{-\frac{1}{\tau_{\mathrm{P}}^j}\delta t}\right) \tag{7}$$

Substituting the control law (Eq. (4)) in the difference equation (Eqs. (6)), the following approximate description of output convergence dynamics is obtained:

$$\begin{bmatrix}\tilde{y}_j(t_{k+1})\\ S_j(t_{k+1})\end{bmatrix} = \begin{bmatrix}\alpha_j - \beta_j K_{\mathrm{C}}^j & -\beta_j\frac{K_{\mathrm{C}}^j}{\tau_{\mathrm{I}}^j}\\ \delta t & 1\end{bmatrix}\begin{bmatrix}\tilde{y}_j(t_k)\\ S_j(t_k)\end{bmatrix} \tag{8}$$

Discrete dynamics are stable (or convergent) if the eigenvalues are inside the unit circle of complex space [22], which depends on the δt, K_{C}^j and τ_{I}^j chosen to fulfill this condition. In Zavala-Guzman et al. [8], the convergence dynamics were matched with a stable second-order ODE

$$\left(\tau_{\mathrm{R}}^j\right)^2\ddot{y}_{\mathrm{R}}^j + 2\xi_{\mathrm{R}}^j\tau_{\mathrm{R}}^j\dot{y}_{\mathrm{R}}^j + y_{\mathrm{R}}^j = 0 \tag{9}$$

of well-known behavior [23]: τ_{R}^j is the natural time and ξ_{R}^j is the damping factor of reference, whose eigenvalues are:

$$\begin{aligned}\lambda_1^{j*}\left(\tau_{\mathrm{R}}^j, \xi_{\mathrm{R}}^j\right) &= -\frac{\xi_{\mathrm{R}}^j}{\tau_{\mathrm{R}}^j} + \frac{\sqrt{\left(\xi_{\mathrm{R}}^j\right)^2 - 1}}{\tau_{\mathrm{R}}^j},\\ \lambda_2^{j*}\left(\tau_{\mathrm{R}}^j, \xi_{\mathrm{R}}^j\right) &= -\frac{\xi_{\mathrm{R}}^j}{\tau_{\mathrm{R}}^j} - \frac{\sqrt{\left(\xi_{\mathrm{R}}^j\right)^2 - 1}}{\tau_{\mathrm{R}}^j}\end{aligned} \tag{10}$$

Here, the behavior of discrete dynamics (Eq. (8)) is also matched with Eq. (9). The eigenvalues of a time-discretized version of reference dynamics (Eq. (9)), are directly found in [22]:

$$\gamma_1^{j*}(\tau_{\mathrm{R}}^j, \xi_{\mathrm{R}}^j, \delta t) = e^{\lambda_1^{j*}\delta t}, \quad l = 1,\ 2 \tag{11}$$

and its corresponding characteristic polynomial can be obtained

$$\begin{aligned}&\left(\gamma - \gamma_1^{j*}\right)\left(\gamma - \gamma_2^{j*}\right) = 0\\ \text{or}\quad &\left(\gamma_{\mathrm{R}}^j\right)^2 - \left(\gamma_1^{j*} + \gamma_2^{j*}\right)\gamma_{\mathrm{R}}^j + \left(\gamma_1^{j*}\gamma_2^{j*}\right) = 0\end{aligned} \tag{12}$$

Therefore, the characteristic polynomial of convergence dynamics (Eq. (8)) is:

$$\left(\gamma_j\right)^2 - \left(\alpha_j - \beta_j K_{\mathrm{C}}^j + 1\right)\gamma_j + \left(\alpha_j - \beta_j K_{\mathrm{C}}^j + \beta_j\delta t\frac{K_{\mathrm{C}}^j}{\tau_{\mathrm{I}}^j}\right) = 0 \tag{13}$$

Then, the possibility of matching the behavior (Eq. (8)) with Eq. (9), by matching coefficients of Eq. (13) with the ones of Eq. (12), is set out.

Based on the continuous case [8], the natural reference time τ_{R}^j is expressed in terms of the time constant of the process τ_{P}^j, as follows:

$$\tau_{\mathrm{R}}^j = \frac{1}{n}\xi_{\mathrm{R}}^j\tau_{\mathrm{P}}^j \tag{14}$$

where n is the number of times the reference dynamics (Eq. (9)) is faster than the natural response of the output (Eq. (8)).

Next, by implementing Eq. (14) into Eq. (12), and matching coefficients of characteristic polynomials (Eq. (12)) and Eq. (13), the following expressions for controller gains are obtained:

In the case of $\xi_{\mathrm{R}}^j \leq 1$,

$$K_{\mathrm{C}}^j = -\frac{2\left(\alpha_j\left(\delta t, \tau_{\mathrm{P}}^j\right)\right)^n\cos\left(f_{\mathrm{D}}^j\left(\xi_{\mathrm{R}}^j\right)f_{\mathrm{S}}^j\left(\delta t, \tau_{\mathrm{P}}^j\right)n\right) - \alpha_j\left(\delta t, \tau_{\mathrm{P}}^j\right) - 1}{\beta_j\left(\delta t, K_{\mathrm{P}}^j, \tau_{\mathrm{P}}^j\right)} \tag{15}$$

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$$\tau_{\mathrm{I}}^{j} = \frac{2\left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{n}\cos\left(f_{\mathrm{D}}^{j}\left(\xi_{\mathrm{R}}^{j}\right)f_{\mathrm{S}}^{j}\left(\delta t,\tau_{\mathrm{P}}^{j}\right)n\right) - \alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right) - 1}{2\left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{n}\cos\left(f_{\mathrm{D}}^{j}\left(\xi_{\mathrm{R}}^{j}\right)f_{\mathrm{S}}^{j}\left(\delta t,\tau_{\mathrm{P}}^{j}\right)n\right) - \left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{2n} - 1}\delta t \tag{16}$$

$$f_{\mathrm{D}}^{j} = \frac{\sqrt{1-\left(\xi_{\mathrm{R}}^{j}\right)^{2}}}{\xi_{\mathrm{R}}^{j}} \tag{17}$$

$$f_{\mathrm{S}}^{j} = \frac{\delta t}{\tau_{\mathrm{P}}^{j}} \tag{18}$$

In the case of $\xi_{\mathrm{R}}^{j} \geq 1$,

$$K_{\mathrm{C}}^{j} = -\frac{2\left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{n}\cosh\left(f_{\mathrm{D}}^{j}\left(\xi_{\mathrm{R}}^{j}\right)f_{\mathrm{S}}^{j}\left(\delta t,\tau_{\mathrm{P}}^{j}\right)n\right) - \alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right) - 1}{\beta_j\left(\delta t,K_{\mathrm{P}}^{j},\tau_{\mathrm{P}}^{j}\right)} \tag{19}$$

$$\tau_{\mathrm{I}}^{j} = \frac{2\left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{n}\cosh\left(f_{\mathrm{D}}^{j}\left(\xi_{\mathrm{R}}^{j}\right)f_{\mathrm{S}}^{j}\left(\delta t,\tau_{\mathrm{P}}^{j}\right)n\right) - \alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right) - 1}{2\left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{n}\cosh\left(f_{\mathrm{D}}^{j}\left(\xi_{\mathrm{R}}^{j}\right)f_{\mathrm{S}}^{j}\left(\delta t,\tau_{\mathrm{P}}^{j}\right)n\right) - \left(\alpha_j\left(\delta t,\tau_{\mathrm{P}}^{j}\right)\right)^{2n} - 1}\delta t \tag{20}$$

$$f_{\mathrm{D}}^{j} = \frac{\sqrt{\left(\xi_{\mathrm{R}}^{j}\right)^{2}-1}}{\xi_{\mathrm{R}}^{j}} \tag{21}$$

$$f_{\mathrm{S}}^{j} = \frac{\delta t}{\tau_{\mathrm{P}}^{j}} \tag{22}$$

3.1 Systematic Procedure of Tuning

On the basis of tuning expressions (Eqs. (15)–(22)), the following procedure for tuning each controller is outlined below. It is important to mention that the four control loops are tuned simultaneously. For each input-output pair in the tuning procedure:

1. Identify open-loop response parameters: τ_{P}^{j} and K_{P}^{j}.
2. Set the sampling time δt, which depends on the features of the measurement system.
3. Set the damping factor ξ_{R}^{j}. As indicated in Zavala-Guzmán et al. [8], setting ξ_{R}^{j} can follow different criteria; a value of 0.79 was used because it corresponds to a behavior in which the overshoot of reference dynamics (Eq. (9)) does not exceed 2 %.
4. Set a value of n. For example, in a first trial, set $n = 1$.
5. Apply the tuning relations (Eqs. (15)–(18) or (19)–(22)) to calculate controller gains of K_{C}^{j} and τ_{I}^{j}.
6. Test the performance of controllers.
7. If greater convergence rate is desired, and/or feasibilty according practical constraints, repeat from step 4 with a greater value of n.

In summary: once the dynamics parameters for each input-output pair are identified, and the sampling time and damping factors are settled, all controllers can be simultaneously adjusted by only one tuning button: n.

4 Control System Performance

To test the performance that provides the tuning technique, the DWDC control configuration from Ling and Luyben [10] was used, as well as from Zavala-Guzman et al. [8]. The implementation in Aspen Dynamics is schematized in Fig. 3.

The following parameters were applied for this DWDC: $N = 46$, $N_{\mathrm{DS}} = 10$, $N_{\mathrm{DI}} = 33$, $N_{\mathrm{F}} = 21$, $N_{\mathrm{M}} = 20$, $F_{\mathrm{F}} = 1\,\mathrm{kmol\,s^{-1}}$ at $T_{\mathrm{F}} = 358\,\mathrm{K}$, $R = 2.84$, $S_{\mathrm{L}} = 0.353$, $F_{\mathrm{S}} = 0.296\,\mathrm{kmol\,s^{-1}}$, and $Q = 35.69\,\mathrm{MW}$. A pressure drop of 0.0068 atm for each valve tray was assumed, and rigorous hydraulics were considered. The components and composition of the ternary mixture fed to the DWDC and compositions of output streams are given in Tab. 1. The above-mentioned operating conditions were assumed to be the nominal ones. Subseqently, the control system test is to maintain compositions of output streams, specifically the corresponding impurity (x_{D}, y_{P10}, x_{S}, x_{B}), in their nominal values by rejecting changes to the flow and/or composition of the input stream.

The basic parameters of process dynamics for controller tuning (τ_{P}^{j}, K_{P}^{j}) are given in Tab. 2, they were just recalled from

Table 1. Components and composition of DWDC streams.

Components	Composition [mol mol^{-1}]				
	Feed stream	Distillate stream	Side stream	Bottom stream	Stage 1 of prefractionator
Benzene	0.3	0.99	0.001	0	0.543
Toluene	0.3	0.01	0.99	0.01	0.453
o-Xylene	0.4	0	0.009	0.99	0.004

Table 2. Parameters for controller tuning relationships.

Controlled variable	Manipulated variable	Static gain K_{P}^{j} [% %$^{-1}$]	Time constant τ_{P}^{j} [h]
$x_{\mathrm{D(T)}}$	R	90.7	2.9873
$y_{\mathrm{P10(X)}}$	S_{L}	34.5	0.9803
$x_{\mathrm{S(X)}}$	F_{S}	42	2.4703
$x_{\mathrm{B(T)}}$	Q	85	1.9536

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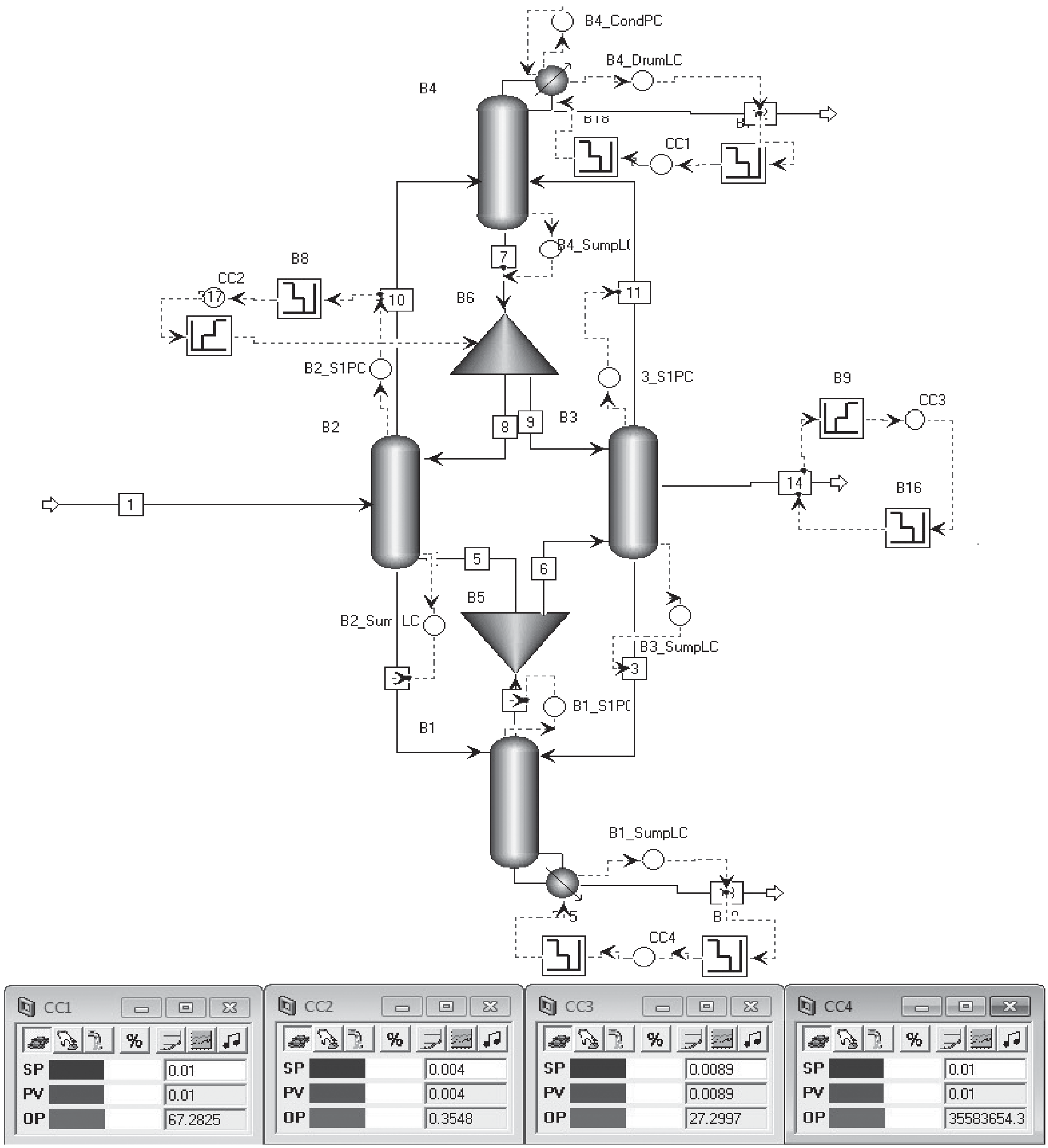

Figure 3. DWDC implementation in Aspen Dynamics®.

Zavala-Guzmán et al. [8]. According to the tuning relationships (Eqs. (15)–(22)), the freedom degrees on tuning are n, ξ, and δt, set on the basis of the measurement technique of output stream compositions. For all following tests and for every controller, ξ was set to 0.79 because it provides the best tradeoff between rise time and overshoot in a second-order ODE, which is given in terms of an unitary time constant and any damping factor.

In the first instance, tests were carried out for regulating the distillation column for several cases of discrete measurements, with controller gains provided by the tuning technique found in Zavala-Guzman et al. [8]. In these tests, a change in the feed flow was faced that occurs after 2 h of nominal operation: convergence was achieved for the cases in which $\delta t < 5$ min (Fig. 4 a). Fig. 4 b illustrates that the controller suddenly breaks down when δt is increased from 5 to 6 min in the distillate composition. Also, it can be observed how performance deteriorates with increasing sampling time. A similar behavior is also found on the other outputs.

In Figs. 5 and 6, the trajectories of the key impurity compositions of each output stream are compared for the case in which the measurement is continuous and for the discrete case where

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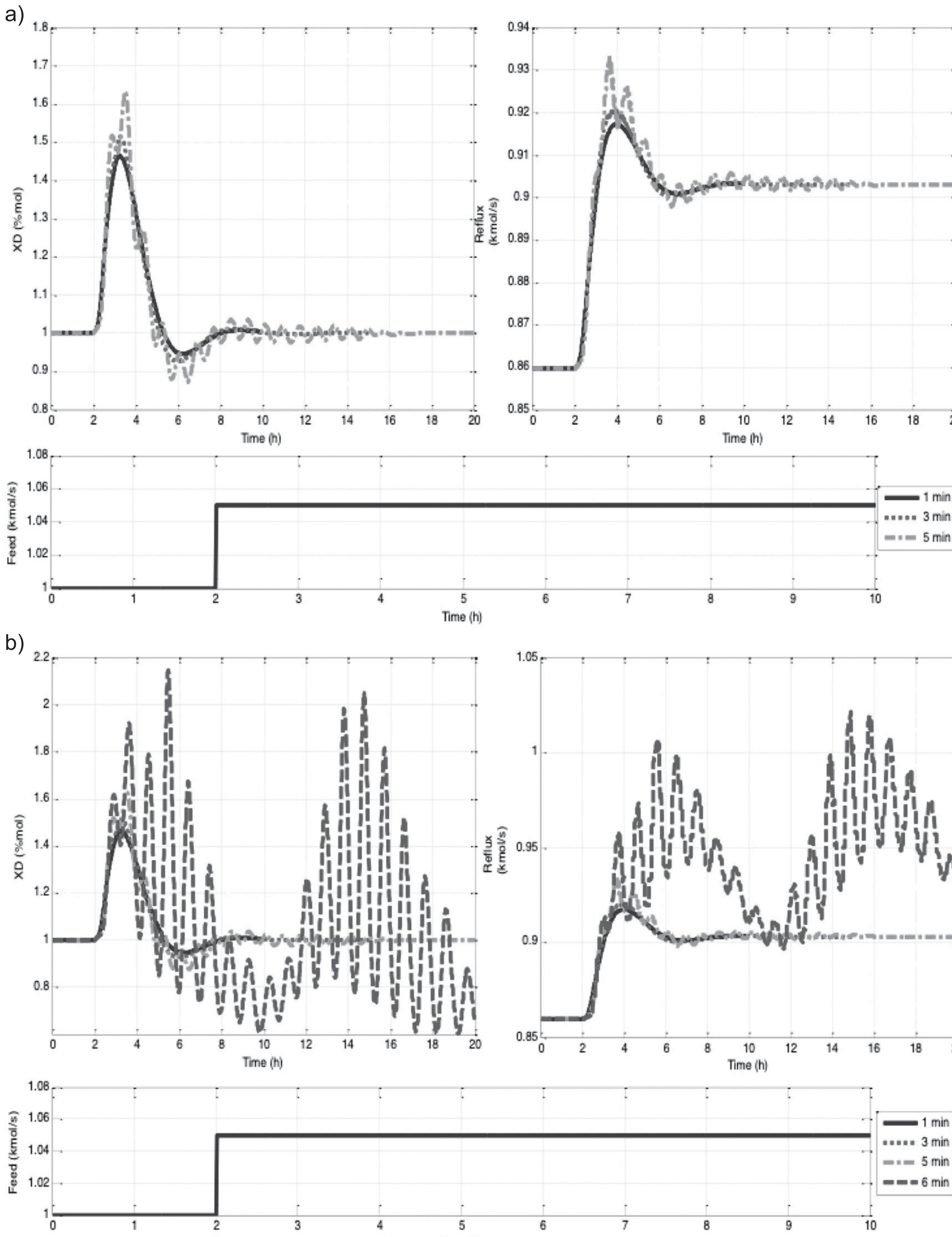

Figure 4. Control system performance with discrete measurements provided by a continuous tuning technique [8]: (a) convergent sampling times, (b) breaking sampling time.

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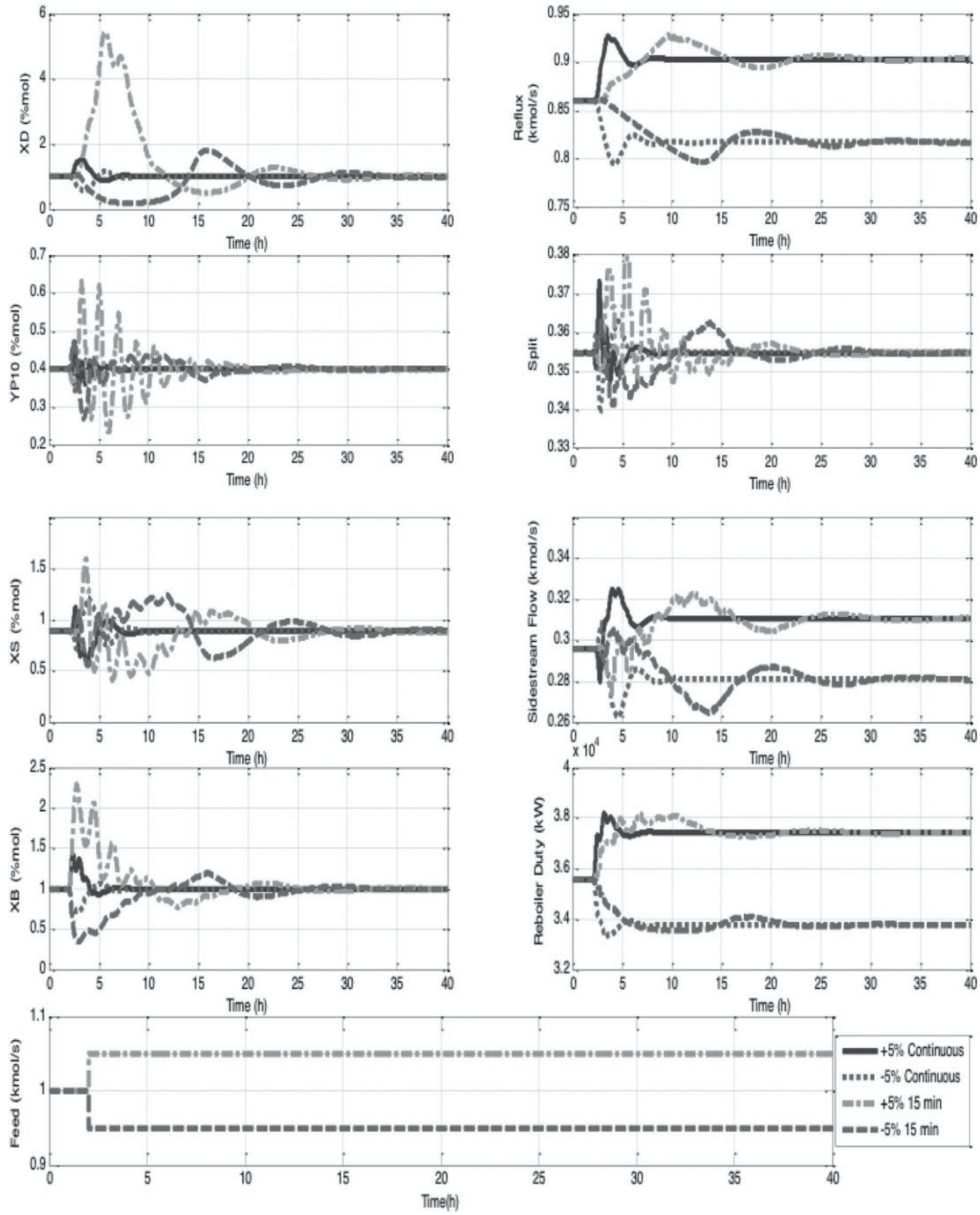

Figure 5. Performance of discrete control system against disturbance in feed flow.

δt = 15 min, whose magnitude order corresponds to a time required for analyzing a sample by a chromatographic technique. The control system faces a change in the feed flow (Fig. 5) and in the composition of the input stream (Fig. 6) that occurs after 2 h of nominal operation. The discrete controllers are tuned with n = 2, and the controllers for continuous signals are tuned with the relationships given in Zavala-Guzman et al. [8], with n = 4. The discrete control system rejects the disturbance, although the sampling interval implies a longer settling time.

In Fig. 7, the behavior of the DWDC control system for several sampling times (δt = 5, 10, 20 min) is demonstrated. For the sake of space, only the trajectories for x_D are illustrated, but similar performance is observed for all other outputs, see Fig. 4. It can be noted that convergence is achieved, although the settling time is increased with respect to the sampling time. The trajectories correspond to the best values of n for each sampling time. The values of n decrease as the sampling time increases.

Fig. 8 depicts the trajectories of the x_D-controller for n = 1, 2, and 3, where δt = 10 min. The disturbance rejection is reached since n = 1, and the higher the n value, the faster the convergence rate, yet the greater the effort in the corresponding con-

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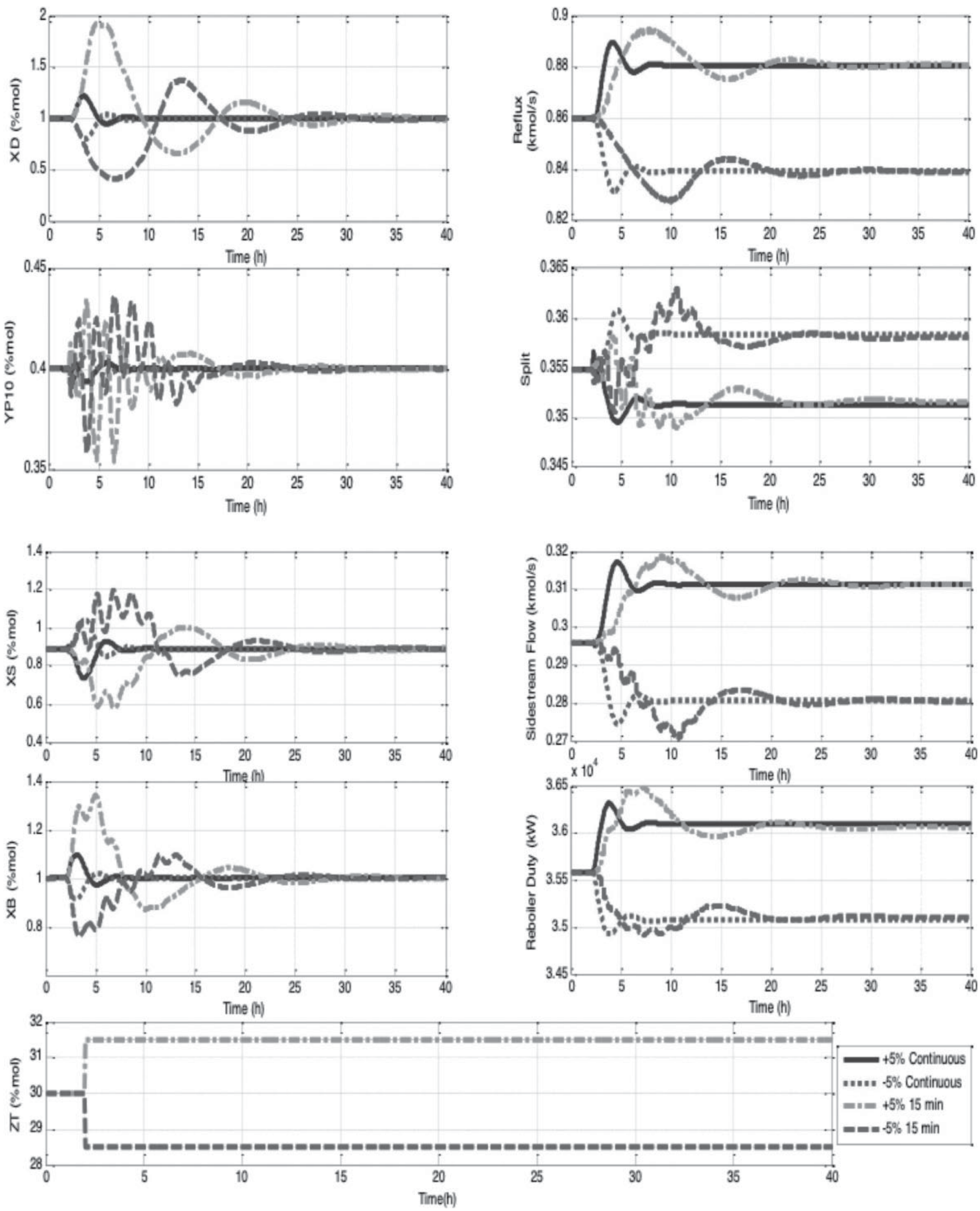

Figure 6. Performance of discrete control system against disturbance in feedstream composition.

trol input. However, the control system breaks down with a large n. Similar behavior is observed for all other outputs.

In other disturbance cases, the same tradeoff of n with respect to δt is detected: to obtain convergence, δt and n cannot be so large. In any case, practical sampling times are below the upper value displayed here. On the other hand, although n is upper-bounded, the control inputs are not associated to saturation problems.

5 Conclusions

A systematic technique to tune PI controllers for a class of DWDCs with periodic discrete measurements was developed. Provided classical parameters of the process dynamics (static gains and time constant of open-loop response) and the sampling-delay time of measurements, this technique determines effective gains for every controller of the DWDC, in a straight-

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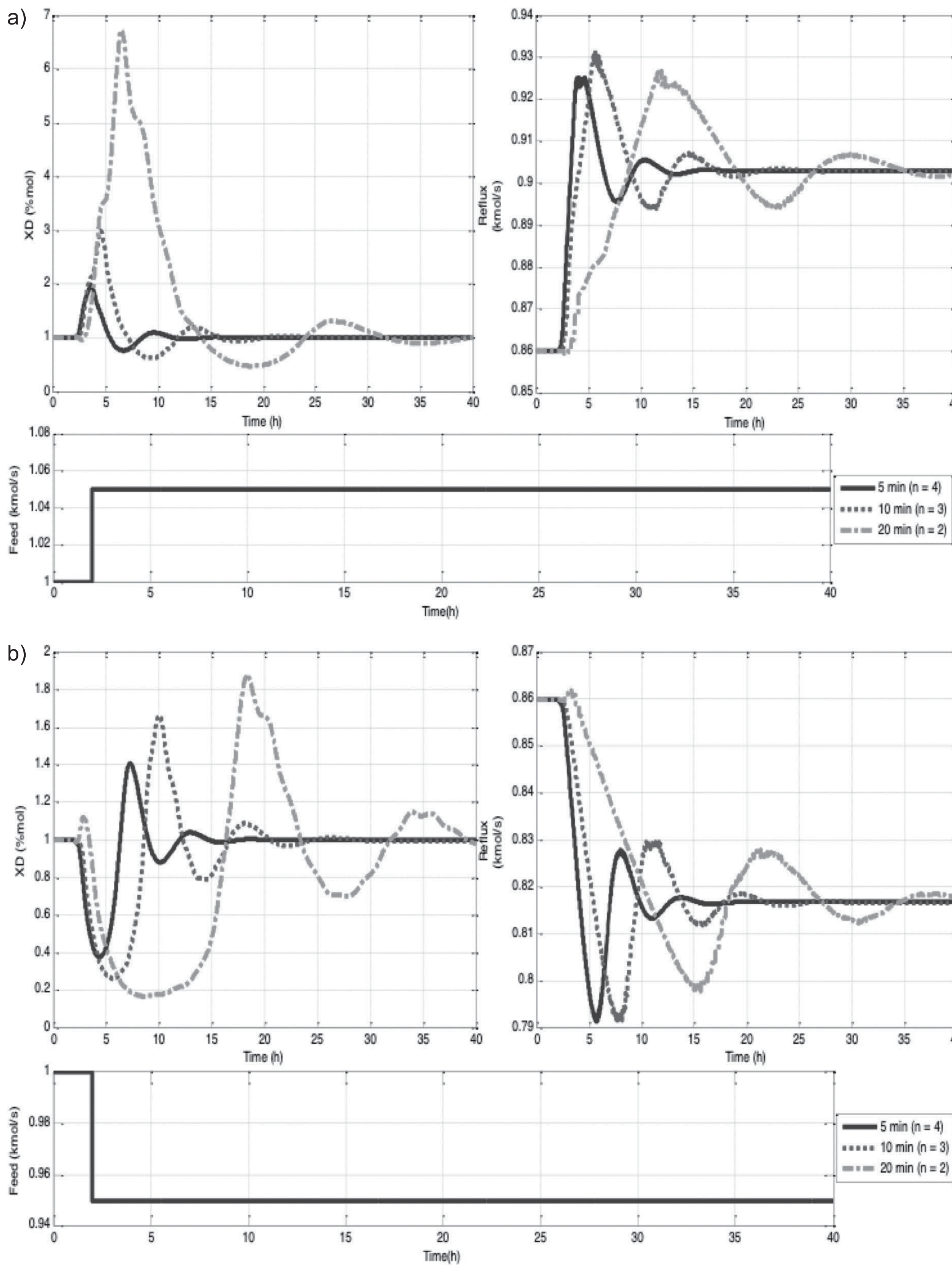

Figure 7. Control system performance against disturbance in the feed flow for different sampling times: (a) positive disturbance, (b) negative disturbance.

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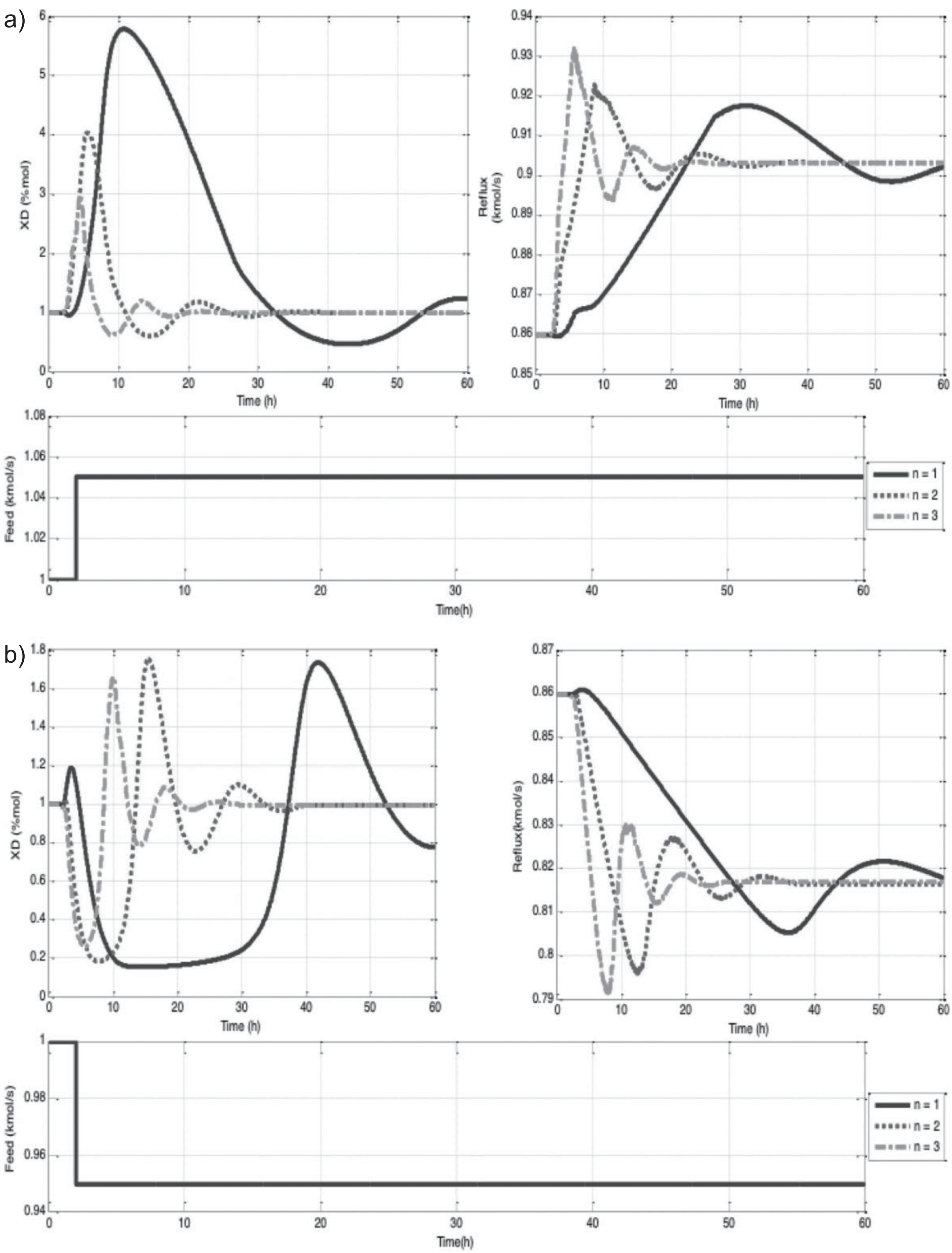

Figure 8. Performance of discrete control system against feed flow disturbance for different values of the tuning parameter *n*: (a) positive disturbance, (b) negative disturbance.

forward and simultaneous way. If a certain adjustment is needed, it is done through a tuning parameter associated with a desired convergence rate, meaning a significant reduction in trial-and-error tuning.

This technique is the discrete counterpart of the one applied for a DWDC with continuous measurement of the composition with good results [8], and it is expected to be applicable for any multiple-input multiple-output (MIMO) control system whose

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dynamics of corresponding input-output pairs are appropriately described by first-order models. The importance of the effective control of a DWDC should also be mentioned, since the energy required in the reboiler depends strongly on the operation around the optimal energy-efficient design.

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Symbols used

F_{F}	[kmol s^{-1}]	feeding rate
F_{S}	[kmol s^{-1}]	side stream flow
K_{C}^{j}	[% %$^{-1}$]	proportional gain of the control loop L_j
K_{P}^{j}	[% %$^{-1}$]	process gain in loop j
L_{i}	[–]	i-th control loop
n	[–]	number of times of the reference dynamics
N	[–]	number of total stages
N_{DI}	[–]	stage where the dividing wall ends
N_{DS}	[–]	stage where the dividing wall starts
N_{F}	[–]	feed stage
N_{M}	[–]	side stream stage
N_{S}	[–]	side stream stage
P_{B}	[atm]	bottom pressure
P_{F}	[atm]	feed pressure
Q	[MW]	reboiler heat duty
R	[–]	reflux ratio
S_{L}	[–]	liquid split ratio
S_{V}	[–]	vapor split ratio
t_k	[s]	k-time
T_{B}	[K]	bottom temperature
T_{F}	[K]	feed temperature
δt	[s]	sampling time
u_j	[–]	j-th manipulated input
x_{B}	[mol mol^{-1}]	mole fraction of the intermediate volatile component in the bottom product
x_{D}	[mol mol^{-1}]	mole fraction of the intermediate volatile component in the distillate product
x_i	[mol mol^{-1}]	mole fraction in the liquid phase
x_{S}	[mol mol^{-1}]	mole fraction of the least volatile component in the side stream product
y_{B}	[mol mol^{-1}]	measurement of x_{B}
y_{D}	[mol mol^{-1}]	measurement of x_{D}
y_j	[–]	j-th controlled variable
$y_{\mathrm{NS-1}}$	[mol mol^{-1}]	mole fraction of the less volatile component at stage $N_{\mathrm{S-1}}$
y_{R}^{j}	[mol mol^{-1}]	measurement of x_{R}
y_{S}	[mol mol^{-1}]	measurement of x_{S}
$\bar{y}$	[–]	nominal value of y
$\tilde{y}$	[–]	deviation value of y
$\dot{\tilde{y}}$	[–]	derivation of the deviation value of y
z_1, z_2, z_3	[mol mol^{-1}]	mole fraction of the first, second, and third component

Greek letters

α_j	[–]	constant in a first order type dynamics
β_j	[–]	constant in a first order type dynamics
γ	[–]	variable in the characteristic polynomial equation
λ	[–]	eigenvalue
ξ_{R}^{j}	[–]	damping factor
τ_{I}^{j}	[h]	integral time of the control loop L_j
τ_{P}^{j}	[s]	natural process time
τ_{R}^{j}	[s]	natural time

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